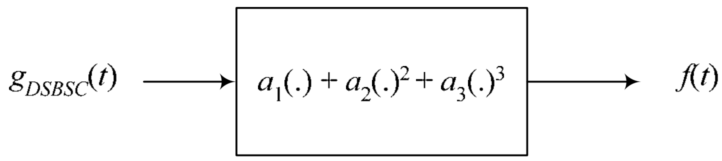
Effect of Non–Linearity on AM and FM signals

Sometimes, the modulated signal after transmission gets distorted due to non–linearities in the channel, for example. In general, when the transmitted modulated signal is affected by channel non–linearity in the channel, the demodulated signal becomes a distorted version of the message signal. We can easily show that the effect non–linearities on different types of amplitude–modulated signals is devastating, while frequency–modulated signals are immune to non–linearities. In fact, the effect of non–linearities on FM signals can be used for generating wideband FM signals from narrowband FM signals, which is an important feature.

<u>*Non–Linearity in AM*</u>: Consider the channel shown below with a DSBSC input signal. The channel is a non–linear channel in which the output signal of the channel is the sum of the input signal and other powers of the input signal.



Let $g_{DSBSC}(t)$ be

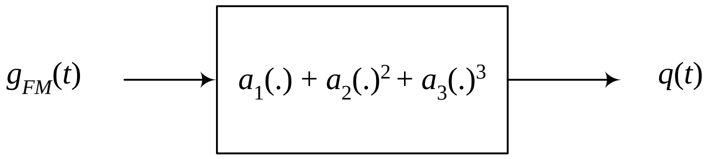
$$g_{DSBSC}(t) = m(t) \cdot \cos\left(\omega_c t\right).$$

The output signal of the channel $f(t)$ becomes

$$\begin{aligned}
f(t) &= a_1 m(t) \cdot \cos\left(\omega_c t\right) + a_2\left[m(t)\cdot\cos\left(\omega_c t\right)\right]^2 + a_3\left[m(t)\cdot\cos\left(\omega_c t\right)\right]^3 \\
&= a_1 m(t)\cdot\cos\left(\omega_c t\right) + a_2 m^2(t)\cdot\cos^2\left(\omega_c t\right) + a_3 m^3(t)\cdot\cos^3\left(\omega_c t\right) \\
&= a_1 m(t)\cdot\cos\left(\omega_c t\right) + \frac{a_2 m^2(t)}{2}\left[1+\cos\left(2\omega_c t\right)\right] + \frac{a_3 m^3(t)}{2}\cdot\cos\left(\omega_c t\right)\left[1+\cos\left(2\omega_c t\right)\right] \\
&= \frac{a_2 m^2(t)}{2} + \left[a_1 m(t) + \frac{a_3 m^3(t)}{2}\right]\cdot\cos\left(\omega_c t\right) + \frac{a_2 m^2(t)}{2}\cos\left(2\omega_c t\right) + \frac{a_3 m^3(t)}{4}\cdot\left[\cos\left(\omega_c t\right)+\cos\left(3\omega_c t\right)\right] \\
&= \underbrace{\frac{a_2 m^2(t)}{2}}_{\text{Around } 0} + \underbrace{\left[a_1 m(t) + \frac{3a_3 m^3(t)}{4}\right]\cdot\cos\left(\omega_c t\right)}_{\text{Around } \omega_c} + \underbrace{\frac{a_2 m^2(t)}{2}\cos\left(2\omega_c t\right)}_{\text{Around } 2\omega_c} + \underbrace{\frac{a_3 m^3(t)}{4}\cdot\cos\left(3\omega_c t\right)}_{\text{Around } 3\omega_c}
\end{aligned}$$

So, unless a_3 is zero, it is clear that the original modulated signal $g_{DSBSC}(t)$ given above cannot be extracted from the received signal $f(t)$ because the terms with frequency around ω_c does not contain only $m(t)$ but also $m^3(t)$. So, DSBSC (and AM modulation in general) is vulnerable to non–linearities.

<u>*Non–Linearity in FM*</u>: Again, consider the same channel shown given above with an FM input signal.



Let $g_{FM}(t)$ be

$$g_{FM}(t) = A\cos\left(\omega_c t + k_f \int_{-\infty}^{\infty} m(\alpha)d\alpha\right).$$

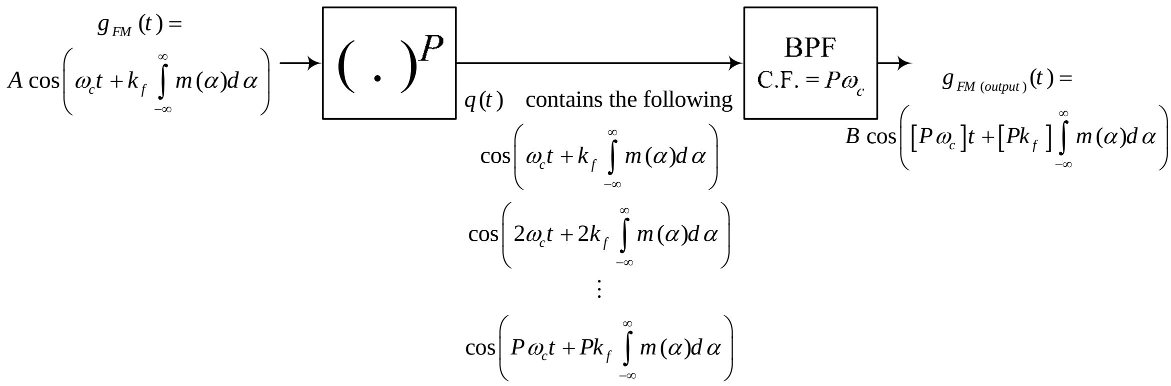
The output signal of the channel $q(t)$ becomes

$$\begin{aligned}
q(t) &= a_1 A\cos\left(\omega_c t + k_f \int_{-\infty}^{\infty} m(\alpha)d\alpha\right) + a_2\left[A\cos\left(\omega_c t + k_f \int_{-\infty}^{\infty} m(\alpha)d\alpha\right)\right]^2 \\
&\quad + a_3\left[A\cos\left(\omega_c t + k_f \int_{-\infty}^{\infty} m(\alpha)d\alpha\right)\right]^3 \\
&= a_1 A\cos\left(\omega_c t + k_f \int_{-\infty}^{\infty} m(\alpha)d\alpha\right) + \frac{a_2 A}{2}\left[1+\cos\left(2\omega_c t + 2k_f \int_{-\infty}^{\infty} m(\alpha)d\alpha\right)\right] \\
&\quad + \frac{a_3 A}{2}\left[1+\cos\left(2\omega_c t + 2k_f \int_{-\infty}^{\infty} m(\alpha)d\alpha\right)\right]\cos\left(\omega_c t + k_f \int_{-\infty}^{\infty} m(\alpha)d\alpha\right) \\
&= \underbrace{\frac{a_2 A}{2}}_{DC} + \underbrace{\left[a_1 A + \frac{3a_3 A}{4}\right]\cos\left(\omega_c t + k_f \int_{-\infty}^{\infty} m(\alpha)d\alpha\right)}_{\text{Around } \omega_c \text{ with } k_f' = k_f} + \underbrace{\frac{a_2 A}{2}\cos\left(2\omega_c t + 2k_f \int_{-\infty}^{\infty} m(\alpha)d\alpha\right)}_{\text{Around } 2\omega_c \text{ with } k_f' = 2k_f} \\
&\quad + \underbrace{\frac{a_3 A}{4}\cos\left(3\omega_c t + 3k_f \int_{-\infty}^{\infty} m(\alpha)d\alpha\right)}_{\text{Around } 3\omega_c \text{ with } k_f' = 3k_f}
\end{aligned}$$

In this case, it is clear that all the coefficients of the different terms are simply constants. The terms are in fact different FM signals with different frequencies and different values of the parameter k_f. But, the important conclusion is that the original FM signal can easily be extracted from the output signal of the non–linear channel using a filter centered at the carrier frequency and bandwidth equal to the bandwidth of the FM signal. So, FM signals are IMMUNE (does not get damaged) to non–linearities.

Use of Non–linearity for Manipulating FM Signals

The fact that passing an FM signal through a non–linear device results in a set of FM signals with different carrier frequencies and different parameter k_f (and therefore different frequency variation parameter $\Delta\omega$), we can use this process for manipulating the carrier frequency ω_c and/or the frequency variation $\Delta\omega$. In general, passing the FM signal through a non–linear device with a maximum non–linearity power of *P* will give *P* different FM signals as shown in the block diagram below.

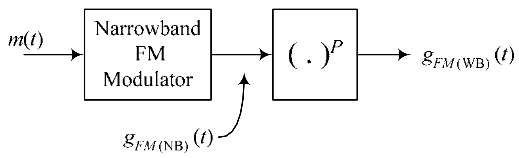


Passing the output of the *P*–power non–linear device through a BPF with center frequency $P\omega_c$ and bandwidth equal to the bandwidth of the last FM signal will extract that signal and reject the other FM signal. This process can be used to obtain wideband FM signals from narrowband FM signals as will be described through a set of examples next. We will assume that the non–linear devices that we use in the following examples have built-in BPFs to eliminate the undesired FM components.

Generation of Wideband FM Signals

Indirect Method for Wideband FM Generation:

Consider the following block diagram



A narrowband FM signal can be generated easily using the block diagram of the narrowband FM modulator that was described in a previous lecture. The narrowband FM modulator generates a narrowband FM signal using simple components such as an integrator (an OpAmp), oscillators, multipliers, and adders. The generated narrowband FM signal can be converted to a wideband FM signal by simply passing it through a non–linear device with power *P*. Both the carrier frequency and the frequency deviation Δf of the narrowband signal are increased by a factor *P*. Sometimes, the desired increase in the carrier frequency and the desired increase in Δf are different. In this case, we increase Δf to the desired value and use a frequency shifter (multiplication by a sinusoid followed by a BPF) to change the carrier frequency to the desired value.

Example 1: A narrowband FM modulator is modulating a message signal $m(t)$ with bandwidth 5 kHz and is producing an FM signal with the following specifications

f_{c1} = 300 kHz,

Δf_1 = 35 Hz.

We would like to use this signal to generate a wideband FM signal with the following specifications

f_{c2} = 135 MHz,

Δf_2 = 77 kHz.

Show the block diagram of several systems that will perform this function and specify the charactestics of each system

Solution: We see that the ratio of the carrier frequencies is

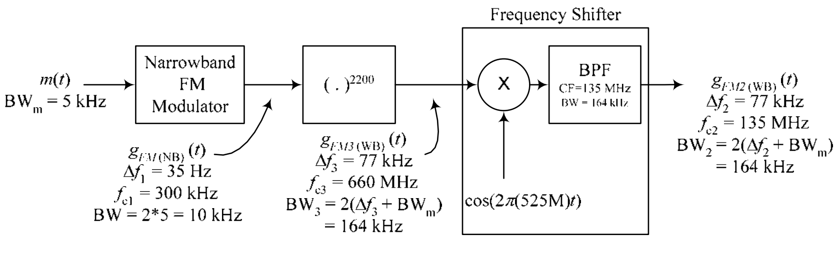
$$\frac{f_{c2}}{f_{c1}} = \frac{135*10^6}{300*10^3} = 450 \,,$$

and the ratio of the frequency variations is

$$\frac{\Delta f_2}{\Delta f_1} = \frac{77*10^3}{35} = 2200 \,.$$

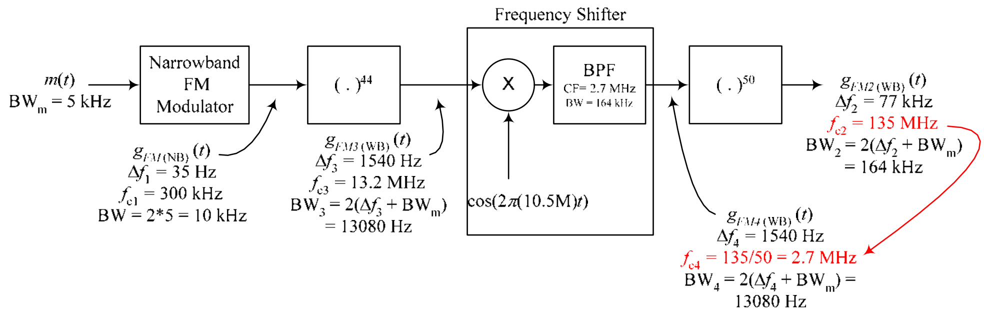
Therefore, we should feed the narrowband FM signal into a single (or multiple) non–linear device with a non–linearity order of $\Delta f_2/\Delta f_1$ = 2200. If we do this, the carrier frequency of narrowband FM signal will also increase by a factor of 2200, which is higher than what is required. This can easily be corrected by frequency shifting. If we feed the narrowband FM signal into a non–device of order f_{c2}/f_{c1}, we will get the correct carrier frequency but the wrong value for Δf. There is not way of correcting the value of Δf for this signal without affecting the carrier frequency.

System 1:



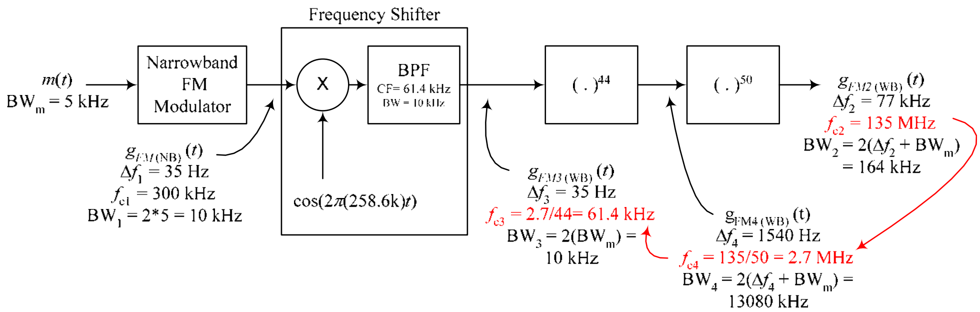
In this system, we are using a single non–linear device with an order of 2200 or multiple devices with a combined order of 2200. It is clear that the output of the non–linear device has the correct Δf but an incorrect carrier frequency which is corrected using a the frequency shifter with an oscillator that has a frequency equal to the difference between the frequency of its input signal and the desired carrier frequency. We could also have used an oscillator with a frequency that is the sum of the frequencies of the input signal and the desired carrier frequency. This system is characterized by having a frequency shifter with an oscillator frequency that is relatively large.

System 2:



In this system, we are using two non–linear devices (or two sets of non–linear devices) with orders 44 and 50 (44*50 = 2200). There are other possibilities for the factorizing 2200 such as 2*1100, 4*550, 8*275, 10*220, … . Depending on the available components, one of these factorizations may be better than the others. In fact, in this case, we could have used the same factorization but put 50 first and 44 second. We want the output signal of the overall system to be as shown in the block diagram above, so we have to insure that the input to the non–device with order 50 has the correct carrier frequency such that its output has a carrier frequency of 135 MHz. This is done by dividing the desired output carrier frequency by the non–linearity order of 50, which gives 2.7 Mhz. This allows us to figure out the frequency of the required oscillator which will be in this case either 13.2–2.7 = 10.5 MHz or 13.2+2.7 = 15.9 MHz. We are generally free to choose which ever we like unless the available components dictate the use of one of them and not the other. Comparing this system with System 1 shows that the frequency of the oscillator that is required here is significantly lower (10.5 MHz compared to 525 MHz), which is generally an advantage.

System 3:



We also can bring the frequency shifter before all the non–linear devices and therefore reduce the frequency of the required oscillator to the minimum value by finding the required carrier frequency at the input of each non–linear device to insure that the carrier frequency of the final output of non–linear devices is the desired final carrier frequency.

<u>For more exercises on this WB FM generation method, refer to the problems at the end of chapter 5 in your textbook.</u>

Direct Method of Generating WB FM Signals

This method is simple in the sense that it uses a single component: the voltage-controlled oscillator (VCO). As described in the section of **Carrier Acquisition for DSBSC systems**, VCOs are devices that produce a sinusoid with a frequency that is proportional to the input signal. So, if the input signal to a VCO is the message signal, the output of the VCO will be an FM modulated signal of the message signal since the frequency of this FM signal changes according to the input message signal.